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Superconformal D-Term Inflation

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Abstract

We study models of hybrid inflation in the framework of supergravity with superconformal matter. F-term hybrid inflation is not viable since the inflaton acquires a large tachyonic mass. On the contrary, D-term hybrid inflation can successfully account for the amplitude of the primordial power spectrum. It is a two-field inflation model which, depending on parameters, yields values of the scalar spectral index down to $n_s \simeq 0.96$. Generically, there is a tension between a small spectral index and the cosmic string bound albeit, within 2σ uncertainty, the current observational bounds can be simultaneously fulfilled.

1 Introduction

Hybrid inflation [1] is an attractive mechanism for generating the cosmological density perturbations. It is naturally realized in the framework of grand unified theories (GUTs) and string theories, especially in the form of D-term inflation [2, 3] where the GUT scale emerges via the Fayet-Iliopoulos (FI) term of an anomalous $U(1)$ symmetry. A further important virtue of D-term inflation is that tree-level supergravity corrections to the inflaton mass of order the Hubble parameter are absent.

D-term inflation has been quantitatively analyzed for the canonical Kähler potential as well as for some non-minimal Kähler potentials [4–6]. The value of the inflaton field is typically $\mathcal{O}(M_P)$ and supergravity corrections are therefore important. In addition to the primordial fluctuations of the inflaton field, the cosmic microwave background is significantly affected by the production of cosmic strings at the end of inflation [7]. Generically, it appears difficult to obtain agreement with observational data [8]. In particular the scalar spectral index n_s turns out to be rather large and the gauge coupling is constrained to small values, in conflict with the motivation of implementing D-term hybrid inflation in GUTs.

In this paper we study D-term inflation in the context of superconformal supergravity models [9] which have recently been considered in connection with Higgs inflation [10–12]. These models are motivated by the underlying superconformal symmetry of supergravity, and have several intriguing features. In particular, there is a Jordan frame in which the matter part of the Lagrangian takes a particularly simple form, closely resembling global supersymmetry. In the Einstein frame, supergravity corrections to scalar masses are suppressed by powers of $1/M_P$, and, contrary to canonical supergravity, the scalar potential does not contain factors which grow exponentially at large field values. The superconformal symmetry is broken by fixing the value of the conformal compensator field, which generates the kinetic term of the graviton [9]. As we shall see, a Fayet-Iliopoulos term can be introduced analogously. A further explicit breaking of superconformal symmetry is a holomorphic contribution to the Kähler potential [12]. This turns D-term inflation into a two-field inflation model.

As we have recently shown, the spontaneous breaking of $B-L$, the difference of baryon and lepton number, at the GUT scale can explain the initial conditions of the hot early universe including baryogenesis and dark matter [13, 14]. This analysis was carried out assuming F-term hybrid inflation. As we shall see, F-term hybrid inflation

is inconsistent with superconformal symmetry. On the contrary, D-term inflation can be implemented with superconformal symmetry and can also incorporate spontaneous $B-L$ breaking at the GUT scale.

The paper is organized as follows. In Section 2 we review the most important features of superconformal models of inflation and in particular discuss the resulting scalar potentials for F- and D-term hybrid inflation. Section 3 deals with an important special case, namely a single-field scenario which arises if inflation lasted sufficiently long before the onset of the final 50 e-folds. The full two-field inflation model is discussed in Section 4. Our conclusions are presented in Section 5.

2 Superconformal models of inflation

2.1 Supergravity with superconformal matter

An attractive class of supergravity models can be defined by requiring the matter sector and its couplings to supergravity to be invariant under superconformal transformations. The superconformal symmetry is explicitly broken only by the pure supergravity part of the action and the superconformal anomaly. Matter interactions at energies below the Planck mass then obey the superconformal symmetry up to corrections suppressed by inverse powers of the Planck mass and radiative corrections [12].

For these theories there exists a Jordan frame in which the Lagrangian takes a remarkably simple form which closely resembles globally supersymmetric theories. The bosonic part for metric and scalar fields z^α is given by [12]

$$\begin{aligned} \frac{1}{\sqrt{-g_J}} \mathcal{L}_J = & \frac{1}{2} M_{\text{P}}^2 (R_J + 6 \mathcal{A}_\mu \mathcal{A}^\mu) - \frac{1}{6} |z|^2 R_J \\ & - G_{\alpha\bar{\alpha}} g_J^{\mu\nu} \tilde{\nabla}_\mu z^\alpha \tilde{\nabla}_\nu \bar{z}^{\bar{\alpha}} - g \partial_\alpha \Phi (T^a z)^\alpha D_a - V_J . \end{aligned} \quad (1)$$

Here the subscript J indicates quantities in the Jordan frame, $|z|^2 = \delta_{\alpha\bar{\alpha}} z^\alpha \bar{z}^{\bar{\alpha}}$, and $G_{\alpha\bar{\alpha}} = \partial_\alpha \partial_{\bar{\alpha}} \Phi$, with $\partial_\alpha = \partial/\partial z^\alpha$ and $\partial_{\bar{\alpha}} = \partial/\partial \bar{z}^{\bar{\alpha}}$ acting on the so-called frame function

$$\Phi(z, \bar{z}) = -3M_{\text{P}}^2 + |z|^2 , \quad (2)$$

which is the coefficient function of the curvature scalar R_J in Eq. (1). The covariant derivative $\tilde{\nabla}_\mu = \partial_\mu - i \mathcal{A}_\mu - i g A_\mu^a T^a$ contains an auxiliary gauge field \mathcal{A}_μ and the dynamical gauge fields A_μ^a , with T^a being the corresponding generators; D_a are the

auxiliary components of the vector superfields. The scalar potential is determined by the superpotential W and the gauge kinetic function f ,

$$V_J = G^{\alpha\bar{\alpha}} \partial_\alpha W \partial_{\bar{\alpha}} \bar{W} + \frac{1}{2} (\text{Re} f)_{ab} D_a D_b , \quad (3)$$

with $G^{\alpha\bar{\alpha}} = G_{\alpha\bar{\alpha}}^{-1}$; for the frame function (2) one has $G^{\alpha\bar{\alpha}} = \delta^{\alpha\bar{\alpha}}$. Local Weyl invariance requires a cubic superpotential W and a constant gauge kinetic function f . It is remarkable that, up to corrections described by the auxiliary field \mathcal{A}_μ , the matter part of the Lagrangian (1) is that of global supersymmetry [12].

The Lagrangian in the Einstein frame with metric g is obtained by performing the transformation

$$g_{J\mu\nu} = \Omega^2 g_{\mu\nu} , \quad \text{with} \quad \Omega^2 = -\frac{3M_{\text{P}}^2}{\Phi} = \left(1 - \frac{|z|^2}{3M_{\text{P}}^2}\right)^{-1} . \quad (4)$$

Eliminating also the auxiliary vector field \mathcal{A}_μ one obtains, up to a total derivative,

$$\frac{1}{\sqrt{-g}} \mathcal{L} = \frac{1}{2} M_{\text{P}}^2 R - K_{\alpha\bar{\alpha}} g^{\mu\nu} \nabla_\mu z^\alpha \nabla_\nu \bar{z}^{\bar{\alpha}} - \Omega^4 g \partial_\alpha \Phi (T^a z)^\alpha D_a - V , \quad (5)$$

with $K_{\alpha\bar{\alpha}} = \partial_\alpha \partial_{\bar{\alpha}} K$ and Kähler potential

$$K(z, \bar{z}) = -3M_{\text{P}}^2 \ln \left(-\frac{1}{3M_{\text{P}}^2} \Phi(z, \bar{z}) \right) . \quad (6)$$

Note that the covariant derivative $\nabla_\mu = \partial_\mu - ig A_\mu^a T^a$ does not contain the auxiliary field \mathcal{A}_μ anymore. For the scalar potential in the Einstein frame one obtains ($f = \delta_{ab}$),

$$\begin{aligned} V &= \Omega^4 \left(G^{\alpha\bar{\alpha}} \partial_\alpha W \partial_{\bar{\alpha}} \bar{W} + \frac{1}{2} D_a^2 \right) \\ &= e^{K/3M_{\text{P}}^2} \left(K^{\alpha\bar{\alpha}} \mathcal{D}_\alpha W \mathcal{D}_{\bar{\alpha}} \bar{W} - \frac{|W|^2}{3M_{\text{P}}^2} \right) + \frac{1}{2} (\Omega^2 D_a)^2 , \end{aligned} \quad (7)$$

where $K^{\alpha\bar{\alpha}} = K_{\alpha\bar{\alpha}}^{-1}$ is the inverse Kähler metric, and $\mathcal{D}_\alpha = \partial_\alpha + \partial_\alpha K / M_{\text{P}}^2$.

2.2 Breaking superconformal symmetry

In the Jordan frame Lagrangian (1) superconformal symmetry is explicitly broken by the kinetic term of the gravitational field. Full superconformal symmetry can be achieved by introducing a compensator field z^0 and replacing the frame function by the $SU(1, n)$ invariant real function

$$\Xi(z^0, \bar{z}^0; z, \bar{z}) = -|z^0|^2 + |z|^2 . \quad (8)$$

The choice $z^0 = \sqrt{3}M_{\text{P}}$, which corresponds to fixing a gauge for the local conformal symmetry, then yields the frame function,

$$\Xi(z^0, \bar{z}^0; z, \bar{z})|_{z^0=\sqrt{3}M_{\text{P}}} = \Phi(z, \bar{z}) . \quad (9)$$

As suggested in [12], given a gauge singlet $\chi_{\alpha\beta} z^\alpha z^\beta$ with $\chi_{\alpha\beta}$ dimensionless, superconformal symmetry can be explicitly broken by using instead of (8) the real function

$$\Xi(z^0, \bar{z}^0; z, \bar{z}) = -|z^0|^2 + |z|^2 + \left(\chi_{\alpha\beta} \frac{z^\alpha z^\beta \bar{z}^0}{z^0} + \text{h.c.} \right) . \quad (10)$$

After gauge fixing one obtains the modified frame function

$$\Phi(z, \bar{z}) = -3M_{\text{P}}^2 + |z|^2 + J(z) + \bar{J}(\bar{z}) , \quad J(z) = \chi_{\alpha\beta} z^\alpha z^\beta , \quad (11)$$

corresponding to a Weyl rescaling between Jordan and Einstein frame with

$$\Omega = \left(1 - \frac{1}{3M_{\text{P}}^2} (|z|^2 + J(z) + \bar{J}(\bar{z})) \right)^{-1/2} . \quad (12)$$

In the following analysis the symmetry breaking term $J(z)$ will play an important role. As we shall see, it will turn the familiar single-field D-term inflation model into a two-field inflation model.

We are particularly interested in adding for a $U(1)$ gauge symmetry a FI-term to the Lagrangian (1). Naively, this would correspond to the substitution $g\partial_\alpha\Phi Q z^\alpha D \rightarrow g(\partial_\alpha\Phi Q z^\alpha + \xi) D$, where Q is the charge generator. This, however, would introduce another explicit breaking of superconformal symmetry, since ξ is a constant of mass dimension two. In the Lagrangian (1) superconformal symmetry breaking only arises from $\Xi(z^0, \bar{z}^0; z, \bar{z})$ after gauge fixing. This suggests to add to Eq. (1) a term with dimensionless constant $\hat{\xi}$,

$$\begin{aligned} \Delta\mathcal{L}_J^\xi &= g\Xi(z^0, \bar{z}^0; z, \bar{z})|_{z^0=\sqrt{3}M_{\text{P}}} \hat{\xi} D \\ &= g\Omega^{-2} \xi D , \end{aligned} \quad (13)$$

where $\xi = -3M_{\text{P}}^2 \hat{\xi}$ has mass dimension two. Note that in the Jordan frame the FI-term is field dependent.

Using Eqs. (5) and (7) and eliminating the auxiliary field D , one immediately obtains for the D-term scalar potential in the Einstein frame,

$$\begin{aligned} V &= \frac{g^2}{2} (\Omega^2 \partial_\alpha \Phi Q z^\alpha + \xi)^2 \\ &= \frac{g^2}{2} (\partial_\alpha K Q z^\alpha + \xi)^2 , \end{aligned} \quad (14)$$

which is the standard supergravity expression [15]. Note that from now on we work in the Einstein frame.

The consistency of a constant FI-term in supergravity is a subtle issue [16–20]. We have in mind a field dependent, effectively constant FI-term at the GUT scale, as it can arise in the weakly coupled heterotic string due to the Green-Schwarz mechanism of anomaly cancellation, where $\xi_{\text{GS}} = g_s^2 \text{Tr} Q M_{\text{P}}^2 / (192\pi^2)$ [21]. Here $\text{Tr} Q$ is the sum over $U(1)$ charges and g_s is the string coupling, which depends on the dilaton. Clearly, a GUT scale FI-term requires an appropriate stabilization of the dilaton and other moduli fields (see, for example, Refs. [16, 22–24]). A related problem is the connection between the GUT scale and supersymmetry breaking. A thorough discussion of these important questions goes beyond the scope of the present paper.

2.3 Scalar potential and F-term inflation

Breaking superconformal symmetry by the holomorphic term J in Eq. (11) significantly modifies the scalar potential. From Eq. (6) one obtains for the frame function Φ given in Eq. (11) the Kähler metric

$$K_{\alpha\bar{\alpha}} = \Omega^2 \left(\delta_{\alpha\bar{\alpha}} - \frac{1}{\Phi} \partial_{\alpha} \Phi \partial_{\bar{\alpha}} \Phi \right) . \quad (15)$$

One easily verifies that the inverse Kähler metric is given by

$$K^{\alpha\bar{\alpha}} = \Omega^{-2} \left(\delta^{\alpha\bar{\alpha}} + \frac{1}{\Delta} \delta^{\alpha\bar{\beta}} \partial_{\bar{\beta}} \Phi \delta^{\beta\bar{\alpha}} \partial_{\beta} \Phi \right) , \quad (16)$$

where

$$\Delta = \Phi - \delta^{\alpha\bar{\alpha}} \partial_{\alpha} \Phi \partial_{\bar{\alpha}} \Phi . \quad (17)$$

Inserting Eq. (16) into the expression (7), one obtains for the F-term scalar potential in the Einstein frame the compact expression¹

$$V_F = \Omega^4 \left(\delta^{\alpha\bar{\alpha}} \partial_{\alpha} W \partial_{\bar{\alpha}} \bar{W} + \frac{1}{\Delta} |\delta^{\alpha\bar{\alpha}} \partial_{\alpha} W \partial_{\bar{\alpha}} \Phi - 3W|^2 \right) . \quad (18)$$

Clearly, for superpotentials cubic in the fields and $J = 0$, the second term in the bracket vanishes and one obtains the F-term potential of global supersymmetry up to the rescaling factor Ω^2 between Jordan and Einstein frame.

¹During the preparation of this paper, Ref. [25] appeared where the same expression for the F-term potential has been found.

The expression (18) holds for all superpotentials and it is instructive to apply it to the superpotential of F-term hybrid inflation [26–29],

$$W = \lambda S (\phi_+ \phi_- - v^2) . \quad (19)$$

Here ϕ_{\pm} are ‘waterfall’ fields, v is a mass parameter and S contains the inflaton; the coupling λ is chosen to be real.

F-term hybrid inflation typically yields a scalar spectral index which is too large compared to observation. One may hope to improve the situation by a proper choice of the χ -parameter of the frame function

$$\Phi = -3M_{\text{P}}^3 + |S|^2 + |\phi_-|^2 + |\phi_+|^2 + \frac{\chi}{2} (S^2 + \bar{S}^2) , \quad (20)$$

where we have used the same symbols for chiral superfields and their scalar components; the parameter χ is chosen to be real. This yields a non-minimal coupling of the inflaton field to gravity. From Eq. (18) one then obtains for the scalar potential

$$V_F = \lambda^2 \left(|S|^2 (|\phi_+|^2 + |\phi_-|^2) + |\phi_+ \phi_- - v^2|^2 - \frac{|2v^2 S + \chi(\phi_+ \phi_- - v^2) \bar{S}|^2}{3M_{\text{P}}^2 + \frac{\chi}{2}(S^2 + \bar{S}^2) + \frac{\chi^2}{4}|S|^2} \right) . \quad (21)$$

Along the expected inflationary trajectory, i.e., for $\phi_{\pm} = 0$, one has

$$V_F = \lambda^2 v^4 - \frac{\lambda^2 v^4 |2S - \chi \bar{S}|^2}{3M_{\text{P}}^2 + \frac{\chi}{2}(S^2 + \bar{S}^2) + \frac{\chi^2}{4}|S|^2} . \quad (22)$$

Unfortunately, this potential has a large tachyonic mass for S and is therefore not phenomenologically viable.

2.4 D-term inflation

Let us now consider D-term hybrid inflation. It has the attractive feature that in string compactifications an FI-term of GUT scale size naturally arises, which is welcome for hybrid inflation. The superpotential reads

$$W = \lambda S \phi_+ \phi_- , \quad (23)$$

and the frame function is again given by Eq. (20). The corresponding F-term scalar potential reads

$$V_F = \Omega^4 \lambda^2 \left(|S|^2 (|\phi_+|^2 + |\phi_-|^2) + |\phi_+ \phi_-|^2 - \frac{\chi^2 |\phi_+|^2 |\phi_-|^2 |S|^2}{3M_{\text{P}}^2 + \frac{1}{2}\chi(S^2 + \bar{S}^2) + \chi^2 |S|^2} \right) . \quad (24)$$

This expression agrees with the potential (21) in the case $v = 0$. For field values below the Planck mass the potential (24) is well behaved. The potential vanishes identically for $\phi_{\pm} = 0$, which corresponds to the inflationary trajectory.

The potential (24) is supplemented by a D-term scalar potential of a $U(1)$ gauge interaction under which the chiral superfields S and ϕ_{\pm} have charge 0 and $\pm q$, respectively. The corresponding scalar potential with nonvanishing FI-term is given by

$$V_D = \frac{g^2}{2} (\Omega^2 q (|\phi_+|^2 - |\phi_-|^2) - \xi)^2, \quad (25)$$

where g is the gauge coupling. For $\phi_{\pm} = 0$, V_D provides the vacuum energy $V_0 = g^2 \xi^2 / 2$ which drives inflation.

The slope of the inflaton potential is generated by quantum corrections. Along the inflationary trajectory the Weyl rescaling factor reads

$$\Omega_0 = \Omega|_{\phi_{\pm}=0} = \left(1 - \frac{1}{3M_P^2} \left(|S|^2 + \frac{\chi}{2}(S^2 + \bar{S}^2)\right)\right)^{-1/2}. \quad (26)$$

From Eqs. (7) and (15) one then obtains for the part of the Lagrangian quadratic in ϕ_{\pm} ,

$$\mathcal{L}_m = \Omega_0^2 \partial_{\mu} \phi_{\pm}^* \partial^{\mu} \phi_{\pm} - (\Omega_0^4 \lambda^2 |S|^2 \mp \Omega_0^2 q g^2 \xi) |\phi_{\pm}|^2, \quad (27)$$

from which one reads off the scalar masses

$$m_{\pm}^2 = \Omega_0^2 \lambda^2 |S|^2 \mp q g^2 \xi. \quad (28)$$

For $|S|$ larger than a critical value $|S_c|$, both ϕ_+ and ϕ_- have positive mass terms and are stabilized at zero, thus allowing inflation to proceed in the S direction. At $|S| = |S_c|$, m_+^2 turns negative, triggering a phase transition which gives an expectation value to ϕ_+ and ends inflation. The critical value S_c is determined by the relation

$$\Omega^2(S_c) |S_c|^2 = \frac{q g^2 \xi}{\lambda^2}. \quad (29)$$

Supersymmetry is broken along the inflationary trajectory where one has $V > 0$. Hence, quantum corrections to the tree-level potential do not vanish and one obtains the one-loop correction

$$V_{1l} = \frac{1}{64\pi^2} \text{STr} \left[M^4 \left(\ln \left(\frac{M^2}{Q^2} \right) - \frac{1}{2} \right) \right]. \quad (30)$$

Here STr denotes the supertrace running over all fields with S -dependent masses, i.e., ϕ_{\pm} and their fermionic partners. M is the corresponding mass matrix, and Q is an appropriate renormalization scale which also determines the argument of the running gauge coupling. According to the mass sum rule, the Dirac fermion associated with ϕ_{\pm} has mass

$$m_f^2 = \lambda^2 \Omega_0^2 |S|^2 . \quad (31)$$

Inserting Eqs. (28) and (31) into Eq. (30) and choosing the renormalization scale $Q^2 = g^2 q \xi$, one obtains for the one-loop potential,

$$\begin{aligned} V_{1l} &= \frac{g^4 q^2 \xi^2}{32\pi^2} ((x-1)^2 \ln(x-1) + (x+1)^2 \ln(x+1) - 2x^2 \ln x - 1) \\ &= \frac{g^4 q^2 \xi^2}{16\pi^2} \left(1 + \ln x + \mathcal{O}\left(\frac{1}{x}\right) \right) , \end{aligned} \quad (32)$$

where

$$x = \frac{\lambda^2 \Omega_0^2 |S|^2}{q g^2 \xi} = \frac{\Omega_0^2(S) |S|^2}{\Omega_0^2(S_c) |S_c|^2} . \quad (33)$$

The total potential is given by (cf. Eqs. (24), (32))

$$\begin{aligned} V &= (V_F + V_D + V_{1l}) \Big|_{\phi_{\pm}=0} \\ &= \frac{g^2}{2} \xi^2 \left(1 + \frac{g^2 q^2}{8\pi^2} \left(1 + \ln x + \mathcal{O}\left(\frac{1}{x}\right) \right) \right) . \end{aligned} \quad (34)$$

Note that on the inflationary trajectory one has $|S| > |S_c|$ and $x > 1$.

3 Single-field inflation

3.1 Slow-roll equation of motion

We are now ready to tackle the slow-roll equations of motion for the field S . Note that the inflaton field S is not canonically normalized, which leads to a modification of the standard slow-roll equations.

Expressing the Lagrangian for the field S in terms of real and imaginary components, $S = (\sigma + i\tau)/\sqrt{2}$,

$$\frac{1}{\sqrt{-g}} \mathcal{L} = \frac{1}{2} K_{S\bar{S}}(\sigma, \tau) (\partial_{\mu} \sigma \partial^{\mu} \sigma + \partial_{\mu} \tau \partial^{\mu} \tau) - V(\sigma, \tau) , \quad (35)$$

one obtains the slow-roll equations for the homogeneous fields σ and τ ,

$$3K_{S\bar{S}}H\dot{\sigma} = -\frac{dV_{1l}}{d\sigma}, \quad 3K_{S\bar{S}}H\dot{\tau} = -\frac{dV_{1l}}{d\tau}, \quad (36)$$

where now we have set $M_P = 1$ for convenience. These equations can be written as the standard slow roll equations for an effective potential defined by

$$\frac{dV_{eff}}{d\sigma} = \frac{1}{K_{S\bar{S}}} \frac{dV_{1l}}{d\sigma}, \quad \frac{dV_{eff}}{d\tau} = \frac{1}{K_{S\bar{S}}} \frac{dV_{1l}}{d\tau}. \quad (37)$$

Calculating the second derivatives of the potential V_{eff} with respect to σ and τ , one finds that for $\chi < 0$, the trajectory $\sigma \neq 0, \tau = 0$ yields a viable inflationary trajectory along which $d^2V_{eff}/d\tau^2$ is positive. Hence this trajectory is an attractor for a sufficiently long phase of inflation before the onset of the final 50 e-folds. For $\chi > 0$, the situation is reversed and an equivalent inflationary trajectory corresponds to $\sigma = 0, \tau \neq 0$. For $\chi = 0$, the Lagrangian is independent of the phase of S and the inflaton can be identified as the absolute value of S . In the following we choose $\chi \leq 0$.

In this section we will restrict ourselves to the standard case of ‘one-field’ inflation described above, postponing the discussion of possible two-field inflation to Section 4. Inserting the Kähler metric

$$K_{S\bar{S}}|_{\phi_{\pm}, \tau=0} = \frac{1}{1 - \frac{1}{6}(1+\chi)\sigma^2} \left(1 - \frac{(1+\chi)^2\sigma^2}{6(1 - \frac{1}{6}(1+\chi)\sigma^2)} \right) \quad (38)$$

and the one-loop potential (32) into the slow-roll equation (36), one obtains after integrating from σ_* to σ_f ,

$$3 \ln \left(\frac{1 - \frac{1}{6}(1+\chi)\sigma_*^2}{1 - \frac{1}{6}(1+\chi)\sigma_f^2} \right) - \frac{1}{2}\chi(-\sigma_*^2 + \sigma_f^2) \simeq -\frac{g^2q^2}{4\pi^2}N_*. \quad (39)$$

Here σ_f denotes the value of σ at the end of inflation and σ_* is the value of σ N_* e-folds earlier. Inflation ends when either m_+^2 turns negative ($\sigma_f = \sigma_c$) or when the slow-roll conditions are violated ($\sigma_f = \sigma_\eta$). From Eq. (29) and Eq. (45) with $|\eta| = 1$, one finds

$$\sigma_c^2 = \frac{6g^2q\xi}{3\lambda^2 + (1+\chi)g^2q\xi}, \quad \sigma_\eta^2 \approx \frac{g^2q^2}{4\pi^2}. \quad (40)$$

For small field values, satisfying $|1+\chi|\sigma_*^2/6 \ll 1$, Eq. (39) can be solved analytically, leading to

$$\sigma_*^2 \simeq \sigma_f^2 + \frac{g^2q^2}{2\pi^2}N_*. \quad (41)$$

However, for most of parameter space this is a bad approximation, and one has to solve Eq. (39) numerically.

3.2 Slow-roll parameters

In order to calculate the spectral index and other observables, we need to evaluate the slow-roll parameters

$$\epsilon = \frac{1}{2} \left(\frac{V'(\hat{\sigma})}{V} \right)^2, \quad \eta = \frac{V''(\hat{\sigma})}{V}. \quad (42)$$

Here $\hat{\sigma}$ is the canonically normalized inflaton field which is determined by (cf. Eq. (35))

$$\frac{d\sigma}{d\hat{\sigma}} = \frac{1}{\sqrt{K_{S\bar{S}}}}. \quad (43)$$

On the inflationary trajectory the derivatives of the scalar potential with respect to $\hat{\sigma}$ can be written as

$$\begin{aligned} V'(\hat{\sigma}) &= \frac{dV_{1l}}{d\hat{\sigma}} = \frac{dV_{1l}}{d\sigma} \frac{d\sigma}{d\hat{\sigma}}, \\ V''(\hat{\sigma}) &= \frac{d^2V_{1l}}{d\hat{\sigma}^2} = \frac{d\sigma}{d\hat{\sigma}} \frac{d}{d\sigma} \left(\frac{dV_{1l}}{d\hat{\sigma}} \right), \end{aligned} \quad (44)$$

from which one obtains the slow-roll parameters

$$\begin{aligned} \epsilon &\simeq 2 \left(\frac{g^2 q^2}{8\pi^2} \right)^2 \frac{1}{\sigma^2} \frac{1}{1 + \frac{1}{6}\chi(1 + \chi)\sigma^2}, \\ \eta &\simeq -\frac{g^2 q^2}{4\pi^2} \frac{1}{\sigma^2} \frac{(1 - \frac{1}{6}(1 + \chi)\sigma^2)(1 + \frac{1}{3}\chi(1 + \chi)\sigma^2)}{(1 + \frac{1}{6}\chi(1 + \chi)\sigma^2)^2}. \end{aligned} \quad (45)$$

Note that for $\chi = -1$, one obtains the results for D-term inflation in global supersymmetry.

3.3 Results and discussion

Normalization of the scalar power spectrum and cosmic strings

The normalization condition for the amplitude of the primordial power spectrum and the cosmic string bound represent observational constraints which have to be fulfilled by a viable model. Allowing for a cosmic string contribution to the power spectrum of the primordial fluctuations implies extending the usual six parameter Λ CDM fit to the CMB data by an additional parameter which accounts for the cosmic string contribution. Detailed analyses for Nambu-Goto strings and Abelian Higgs (AH) cosmic strings have been carried out by several groups [30–33]. In the waterfall transition ending D-term hybrid inflation, a local $U(1)$ symmetry is broken and AH cosmic strings may be formed.

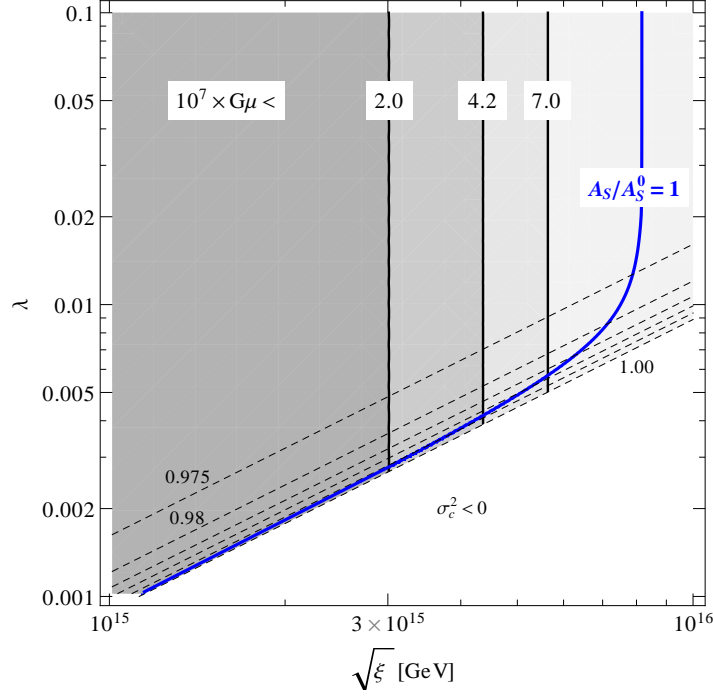


Figure 1: Normalization condition and cosmic string bound for $\chi = -15$, $q = 2$, $g^2 = 1/2$ and $N_* = 50$. The blue line shows the relationship between ξ and λ imposed by the correct normalization of the amplitude of the primordial fluctuations. The black lines denote the cosmic string bound for $G\mu \times 10^7 < 2, 4.2$ and 7 , respectively; the darker shaded regions on the left are in agreement with the constraint. The dashed lines show contours of constant scalar spectral index. The white region to the bottom right must be excluded since there is no positive solution to $m_+^2(\sigma_c) = 0$.

In the following discussion we shall therefore use the results of the recent analysis in Ref. [32] which is based on the field theoretical simulation of cosmic strings in Ref. [34].

The analysis in Ref. [32] yields an upper bound on a cosmic string contribution of about 5% and a best-fit value for the amplitude of the scalar contribution to the primordial fluctuations,

$$A_s^0 = (2.15_{-0.06}^{+0.07}) \times 10^{-9} , \quad (46)$$

where a 1σ error has been given. Comparing this value with the expression calculated in our model,

$$A_s = \frac{1}{12\pi^2} \frac{V^3}{V'^2} \Big|_{\sigma=\sigma_*} , \quad (47)$$

which, using Eq. (32), can be simplified to

$$A_s \simeq \frac{2\pi^2}{3} \frac{\xi^2 \sigma_*^2}{g^2 q^4} \left[1 + \frac{1}{6} \chi (1 + \chi) \sigma_*^2 \right] , \quad (48)$$

one obtains a relation between ξ and λ for given values of χ , q and g .

As an example, we choose $q = 2$ and $g = 1/\sqrt{2}$ in the following, which is motivated by identifying the spontaneously broken $U(1)$ symmetry with $U(1)_{B-L}$ (c.f. [13]). For $\chi = -15$, the implied relation between ξ and λ is represented by the blue line in Fig. 1.

A fit to the CMB data assuming scalar perturbations and AH cosmic strings yields an upper bound on $G\mu$, where $G = (8\pi M_{\text{Pl}}^2)^{-1}$ is Newton's constant,

$$\mu = 2\pi\langle\phi_+\rangle^2 B(\beta) \quad (49)$$

denotes the string tension, and $B(\beta)$ gives the deviation from the Bogomolnyi bound on the string tension [35],

$$B(\beta) = 1.19 \beta^{-0.195}, \quad \text{with} \quad \beta = 2 \frac{m_V^2}{m_\sigma^2}. \quad (50)$$

Inserting the vacuum expectation value of the waterfall field, $\langle\phi_+\rangle = (\xi/q)^{1/2}$, as well as the masses for the $U(1)$ vector boson and the inflaton in the true vacuum, $m_V^2 = m_\sigma^2 = 2g^2 q \xi$, one obtains

$$G\mu = 5.5 \times 10^{-7} \frac{2}{q} \frac{\xi}{(5 \times 10^{15} \text{ GeV})^2}. \quad (51)$$

This is to be compared with the 2σ upper bound found in the analysis of Ref. [32],

$$G\mu < 4.2 \times 10^{-7}. \quad (52)$$

The solid black lines in Fig. 1 correspond to the string tensions $G\mu = (2, 4.2, 7) \times 10^{-7}$. The brighter region on the right of a given line is excluded, whereas the darker region on the left is in agreement with the respective bound.

The upper bounds on the string tension have a considerable theoretical uncertainty. For instance, the upper bounds for Nambu-Goto strings are more restrictive than the ones for AH cosmic strings by about a factor of three [30]. This can be traced back to decay channels into massive radiation for AH cosmic strings [36]. Note also, that all simulations have been done for a bosonic Abelian Higgs model, whereas in D-term inflation one is considering a supersymmetric theory. Additional fermionic decay channels may further relax the cosmic string bound by a factor $\mathcal{O}(1)$. Last but not least, one has to worry about initial conditions. Clearly, strings cannot form until the causal horizon is larger than their characteristic width [34], and one should remember that

tachyonic preheating proceeds very fast. In fact, the expectation value $\langle |\Phi_{\pm}|^2 \rangle$ of the waterfall field grows with time faster than exponentially [37].

Spectral index

With the slow-roll parameters from Eqs. (45) and the value of σ N_* e-folds before the end of inflation, cf. Eq. (39), at hand, we can now easily calculate the spectral index,

$$n_s = (1 - 6\epsilon + 2\eta) \Big|_{\sigma=\sigma_*} . \quad (53)$$

Fig. 2(a) shows the resulting χ dependence for a (ξ, λ) pair compatible with the cosmic string bound and the normalization condition at $\chi = -15$ (cf. Fig. 1). For reference, Fig. 2(b) shows the corresponding χ -dependence of the total amplitude. Both curves are shown over the entire range of allowed χ -values for this choice of ξ and λ , which is bounded from below by the condition that σ_c^2 in Eq. (40) is positive.

The dashed lines show the results obtained using the analytical formulas (45) and (47) with σ_* determined by Eq. (39), the solid lines show the full numerical results. The deviation visible in Fig. 2(a) is due to the approximation of the one-loop potential, which enters in the derivation of Eq. (39) and in the expressions for the slow-roll parameters ϵ and η . To obtain the numerical result, we do not use this approximation, but proceed with the full expression given in the first line of Eq. (32). Note, however, that these corrections only influence the result for the spectral index at the per mille level, proving that the analytical results obtained above do indeed give a good description of the quantitative results.

Throughout the parameter region compatible with the normalization condition and the cosmic string bound, the spectral index is rather high, $n_s \simeq 0.99 - 1.0$. However, taking into account a contribution of cosmic strings close to the current bound significantly modifies the best-fit value of n_s to the CMB data compared to the standard six parameter Λ CDM fit. In Ref. [32], the spectral index matching the amplitude given in Eq. (46) and a cosmic string contribution of about 2% is found to be

$$n_s = 0.969 \pm 0.013 . \quad (54)$$

The obtained values for the spectral index are thus compatible with current observational data at about the 2σ level.

The qualitative behaviour of the relation between the coupling λ and the inflationary energy scale $\sqrt{\xi}$, displayed in Fig. 1, can be easily understood. In the case of small

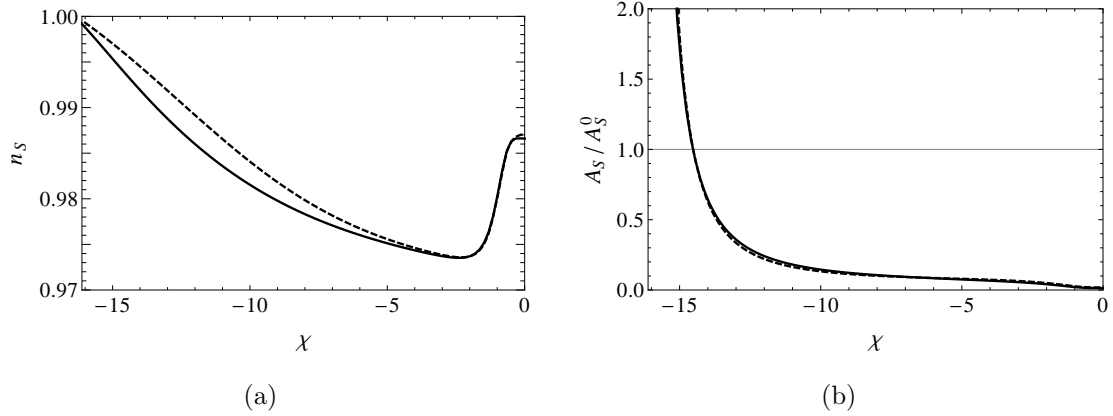


Figure 2: Spectral index and total amplitude for $\sqrt{\xi} = 4.3 \times 10^{15}$ GeV, $\lambda = 4 \times 10^{-3}$, $q = 2$, $g^2 = 1/2$, $N_* = 50$. The solid lines show the numerical results, the dashed lines the analytical ones. The values of ξ and λ are chosen such as to be compatible with the cosmic string bound as well as the normalization constraint for $\chi = -15$ (c.f. Fig. 1).

coupling, $\lambda \lesssim 0.01$, one has $\sigma_*^2 \simeq \sigma_c^2$ (cf. Eq.(40)). The correct fluctuation amplitude is then obtained for small values of $\sqrt{\xi}$ and the cosmic string bound can be satisfied. However, the field value σ_* is large, and one therefore obtains a large spectral index, $n_s \simeq 1$.

On the other hand, for large couplings λ , one has $\sigma_f^2 \ll 1$. For large values of $(-\chi)$, Eq. (39) then implies for the field value σ_* at N_* e-folds,

$$\sigma_*^2 \simeq -\frac{g^2 q^2 N_*}{2\pi^2 \chi} . \quad (55)$$

Interestingly, the amplitude of scalar fluctuations is then only determined by the energy density during inflation, $V_0 = g^2 \xi^2 / 2$ (cf. Eq. (48)),

$$A_s \simeq \frac{V_0}{18\pi^2} N_*^2 . \quad (56)$$

For the spectral index one finds²

$$n_s \simeq (1 - 2\eta)|_{\sigma_*} \simeq 1 - \frac{2}{N_*} \simeq 0.96 . \quad (57)$$

Contrary to the amplitude of scalar fluctuations, the string tension additionally depends on the coupling strength gq (cf. Eq. (51)),

$$G\mu = 5.5 \times 10^{-7} \frac{2\sqrt{2}}{gq} \frac{V_0^{1/2}}{(5 \times 10^{15} \text{ GeV})^2} . \quad (58)$$

²Note the difference to D-term inflation in global supersymmetry, where one has $n_s \simeq 1 - \frac{1}{N_*} \simeq 0.98$, see Ref. [38].

Hence, for large values of $(-\chi)$ and λ , it is always possible to satisfy the cosmic string bound by increasing gq while at the same time keeping n_s small. This is in contrast to the case where $|1 + \chi|\sigma_*^2/6 \ll 1$ and $\sigma_c^2 \ll \sigma_*^2$, with σ_* given by Eq. (41). In this case the amplitude is given by $A_s \simeq 2\epsilon N_*/(3g^2q^2)$ whose value also fixes the string tension. However, increasing gq one moves to a regime of strong coupling and the theoretical consistency of the model becomes questionable.

For the other CMB observables, i.e., the tilt of the spectral index α_s and the tensor to scalar ratio r , we find small values, well within the experimental bounds [39]. For instance, for the parameter point discussed above, $\sqrt{\xi} = 4.3 \times 10^{15}$ GeV, $\lambda = 4 \times 10^{-3}$, $q = 2$, $g^2 = 1/2$, $\chi = -15$ and $N_* = 50$, one obtains

$$\begin{aligned}\alpha_s &= 16\epsilon\eta - 24\epsilon^2 - 2\frac{V'V'''}{V^2}\Big|_{\sigma=\sigma_*} = -1.7 \times 10^{-4}, \\ r &= 16\epsilon\Big|_{\sigma=\sigma_*} = 6.4 \times 10^{-6}.\end{aligned}\tag{59}$$

In conclusion, Fig. 1 shows that there is a considerable region in parameter space, which is compatible with the normalization condition as well as cosmic string bounds. However, for generic gauge coupling strengths gq , this implies a rather large value for the spectral index. Vice versa, in the region of parameter space which yields a spectral index close to the best fit value $n_s \simeq 0.97$, we find a cosmic string tension exceeding the cosmic string bound. In the viable region of parameter space in between these two limiting cases, we thus find a high contribution of cosmic strings close the current bounds as well as a value for the spectral index which is slightly larger than the current best-fit value. Clearly, upcoming experiments will provide further stringent tests of superconformal D-term hybrid inflation.

It is worth stressing that the discussed parameter region allows for large values of the gauge coupling constant g , compatible with grand unification. In this respect, the model presented here differs significantly from D-term inflation with canonical Kähler potential. In the latter case, the masses entering the one-loop potential carry $\exp(|S|^2)$ factors, leading to problems for the super-Planckian values of $|S|$ typically obtained in D-term inflation. Avoiding this forces the gauge coupling g to be small, $g \lesssim 2 \times 10^{-2}$, as found in Ref. [4].

4 Two-field inflation

4.1 Two-field versus single-field inflation

In the previous section, we focused on the situation where one of the two real degrees of freedom of the complex scalar field S plays the role of the inflaton, whereas the value of the other degree of freedom is fixed at zero. This is the case if either the second degree of freedom has a mass of order of the Hubble scale or if inflation before the onset of the final 50 e-folds lasted sufficiently long, so that the inflationary trajectory in the direction of the smallest curvature has become an attractor. Here, with the mass difference between σ and τ governed by the symmetry breaking parameter χ , typically both masses are below the Hubble scale, resulting in a two-field inflation model. This section is hence dedicated to investigating alternative possible trajectories in (σ, τ) field space.

In single-field hybrid inflation, inflation ends at the critical value of the inflaton field, σ_f , determined by the zero point of the mass of the waterfall field, $m_+(\sigma_f) = 0$.³ The starting point σ_* of the inflationary trajectory is determined by solving the slow-roll equation. In two-field inflation, the condition $m_+(\sigma_f, \tau_f) = 0$ defines a line in (σ, τ) field space. From each point on this line $(\sigma_f, \tau_f(\sigma_f))$, a classical inflationary trajectory can be uniquely determined by solving the set of slow-roll equations (36). The resulting trajectory ends at $(\sigma_*(\sigma_f), \tau_*(\sigma_f))$. The single-field case discussed in Section 3 is reproduced for $(\sigma_f, \tau_f) = (\sigma_f^0, 0)$, where σ_f^0 is given by Eq. (40). Hence in two-field inflation, as opposed to single-field inflation, the inflationary predictions are not uniquely determined by the parameters of the Lagrangian, but depend on an additional parameter which labels the various possible trajectories. In the notation above, this additional parameter is σ_f . This is illustrated in Fig. 3.

A generalization of the usual single-field formulas for the amplitude of the scalar fluctuations and the spectral index to the case of multi-field inflation with a non-trivial metric in field space can be found in Ref. [40]. Starting from the action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} h_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right], \quad (60)$$

with $g_{\mu\nu}$ denoting the spacetime metric, h_{ab} the metric on the real scalar field space

³Here and in the following, we assume that the slow-roll conditions hold until the inflaton field reaches its critical value.

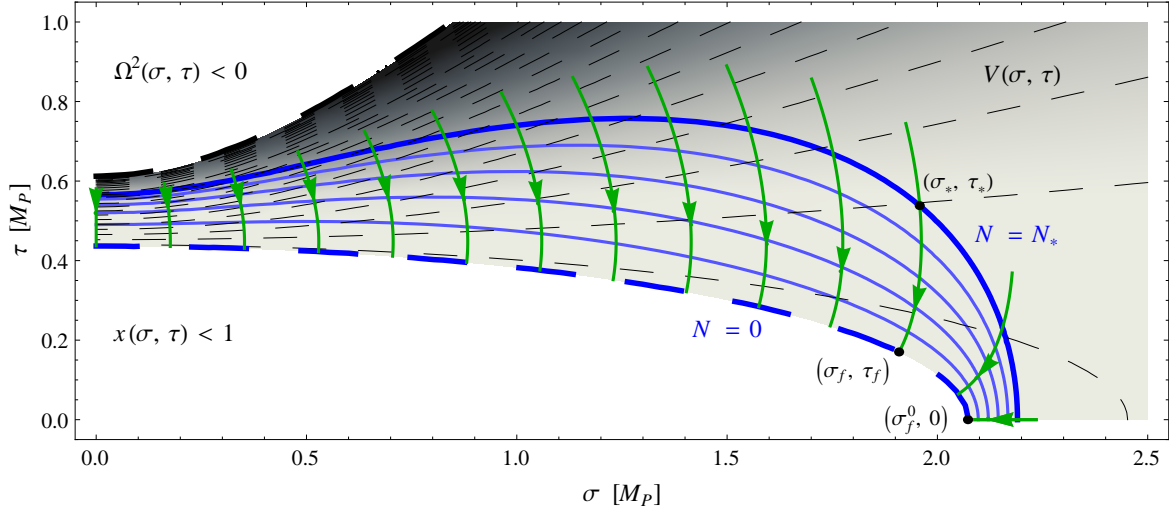


Figure 3: Inflationary trajectories in (σ, τ) field space for $\chi = -15$, $\lambda = 4 \times 10^{-3}$, $\sqrt{\xi} = 4.3 \times 10^{15}$ GeV, $g^2 = 1/2$ and $q = 2$. Contour lines of the scalar potential are denoted by dashed lines. The dashed blue line marks the $m_+ = 0$ condition, the green solid lines show several examples of inflationary trajectories. The blue lines show contours of the number of e-folds N , from $N = 0$ to $N = N_* = 50$. The single field case discussed in Section 3 corresponds to the trajectory coinciding with the σ -axis.

and ϕ^a the real scalar fields of the theory, the slow-roll conditions read

$$(\partial^a V)(\partial_a V) \ll V^2 \quad \text{and} \quad \sqrt{(\nabla^b \partial^a V)(\nabla_b \partial_a V)} \ll V. \quad (61)$$

Here the usual partial derivatives and covariant derivatives in scalar field space are denoted by $\partial_a = \partial/\partial\phi^a$ and $\nabla_a X^b = \partial_a X^b + \Gamma^b_{ca} X^c$. As usual, the metric h_{ab} can be used to raise or lower indices. For inflationary trajectories satisfying these conditions, the authors of Ref. [40] obtain the following expressions for the amplitude of the primordial power spectrum and the spectral index:

$$P_s = \left(\frac{H^2}{2\pi}\right)^2 h^{ab}(\partial_a N)(\partial_b N), \quad (62)$$

$$n_s - 1 = \frac{[2 \nabla_b \partial^a \ln V + (\frac{2}{3} R^a_{bcd} - h^a_b h_{cd}) (\partial^c \ln V)(\partial^d \ln V)] (\partial_a N)(\partial^b N)}{(\partial_e N)(\partial^e N)},$$

with N denoting the number of e-folds, h^{ab} the inverse metric, $h^a_b = \delta^a_b$ and R^a_{bcd} the scalar field space curvature tensor, $R^a_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc} + \Gamma^a_{ce} \Gamma^e_{db} - \Gamma^a_{de} \Gamma^e_{cb}$ with the Christoffel symbols $\Gamma^a_{bc} = \frac{1}{2} h^{ad} (\partial_c h_{db} + \partial_b h_{dc} - \partial_d h_{bc})$.

The number of e-folds N as a function of the scalar fields ϕ^a is determined by integrating along all possible classical trajectories. Each point in field space lies on exactly one classical trajectory. Integrating along this trajectory yields the value of N at this point in field space, which is illustrated by the solid blue contour lines in Fig. 3.

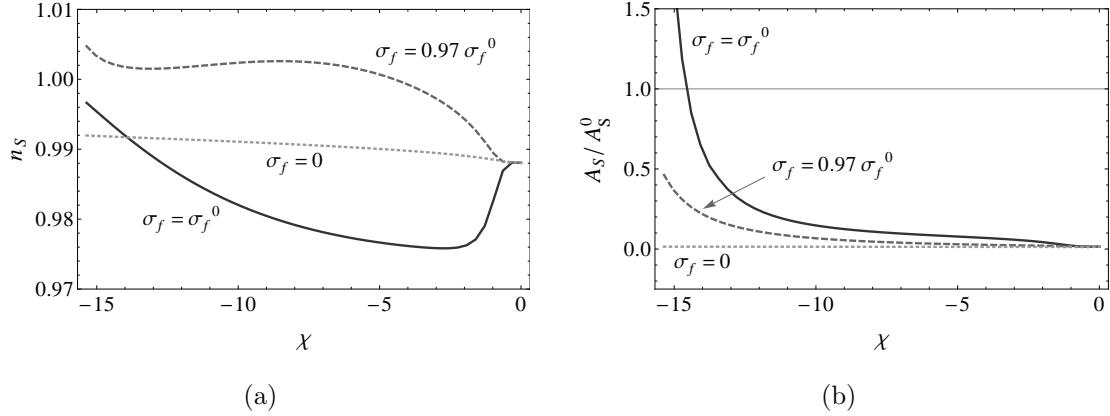


Figure 4: Spectral index and total amplitude resulting from different inflationary trajectories for the same values of model parameters as in the single-field case depicted in Fig. 2, i.e. $\sqrt{\xi} = 4.3 \times 10^{15}$ GeV, $\lambda = 4 \times 10^{-3}$, $q = 2$, $g^2 = 1/2$, $N_* = 50$.

4.2 Two-field results

Fig. 4 shows the spectral index and the amplitude of the scalar power spectrum corresponding to different inflationary trajectories. The solid lines represent the results for the trajectory along the σ -axis, i.e. for $\sigma_f = \sigma_f^0$, hence reproducing the single-field results depicted in Fig. 2. The dotted lines correspond to the other extremal case in which the inflationary trajectory runs along the τ -axis, i.e., in which $\sigma_f = 0$. Finally, the dashed lines show the results for an intermediate trajectory with non-trivial evolution in both σ - and τ -direction.

As illustrated in Fig. 4(b), the amplitude of the scalar power spectrum becomes smaller the more the inflationary trajectory deviates from the σ -axis. Naively, one might expect a different behaviour, since the gradient of N becomes large for inflationary trajectories along the σ - as well as the τ -axis, cf. Fig. 3. But for negative χ the entries of the inverse Kähler metric, $h^{\sigma\sigma} = h^{\tau\tau} = K^{S\bar{S}}$, become increasingly smaller the further one moves along the $N = N_*$ contour away from the σ -axis. As it turns out, this decrease in $K^{S\bar{S}}$ dominates over the change in the gradient of the number of e-folds, so that the amplitude ends up going down as soon as one chooses an inflationary trajectory other than the one discussed in Section 3. In order to understand the decrease in the amplitude more intuitively, it is useful to consult the single-field expression for A_s in Eq. (48). Interpreting V' appearing in this expression as the derivative of the scalar potential along the respective inflationary trajectory, the single-field expression for A_s may serve as a lowest-order approximation of the full multi-field expression in Eq. (62).

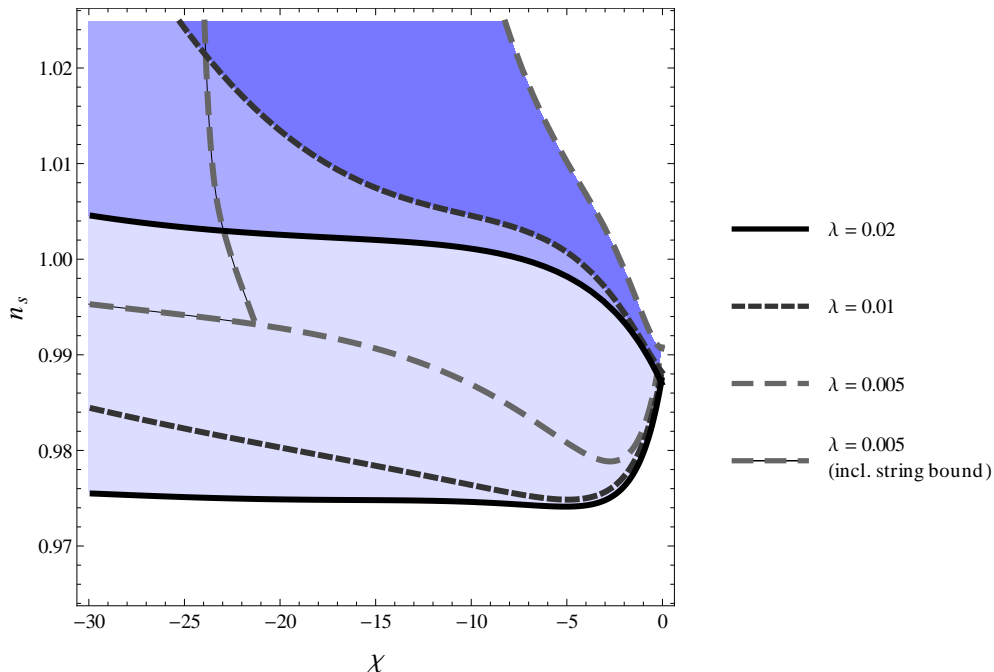


Figure 5: Possible values of the spectral index n_s as a function of χ . The shaded region bounded by a curve with a given stroke style shows the range of possible n_s values achieved by varying the inflationary trajectory for a given value of λ , while constraining the corresponding values of the amplitude to the 3-sigma range of the observed value A_s^0 . For $\lambda = 0.005$, the region to the top left, bounded by the gray solid-dashed curve, is in accordance with the cosmic string bound.

From Eq. (48) it is then apparent that a steeper potential, i.e., a larger V' , entails a smaller amplitude. Since for negative χ the scalar potential indeed becomes steeper the further one moves along the $N = N_*$ contour towards the τ -axis, this explains our observation in Fig. 4(b).

The behaviour of the scalar spectral index n_s is more complicated. We find that typically the minimal value of n_s as a function of χ is enhanced when considering trajectories involving a motion in τ -direction. In the limit $\chi \rightarrow 0$, the three curves for the scalar spectral index as well as the amplitude in Fig. 4 respectively converge to common values. This reflects the fact that for $\chi = 0$ the phase of the complex inflaton field S turns unphysical, rendering all possible trajectories equivalent to each other.

For fixed values of the parameters ξ and λ , the normalization condition, cf. Eq. (46), can be used to eliminate the parameter σ_f , which we introduced in Section 4.1 to distinguish between the different inflationary trajectories. According to Fig. 4, it is for instance possible to find for $\sqrt{\xi} = 4.3 \times 10^{15}$ GeV and $\lambda = 4 \times 10^{-3}$ and for each χ value below $\chi \simeq -14.5$ one particular σ_f , i.e. one inflationary trajectory such that

$A_s = A_s^0$. It is important to note that it is only these sets of parameter values, which are compatible with the normalization condition, that we are allowed to consider when asking for the range of viable n_s values predicted by our model.

In order to determine this range of admissible n_s values, we perform a numerical scan of the parameter space and record n_s for all values of the parameters ξ , λ , χ and σ_f that yield an amplitude A_s within the 3-sigma range of the best-fit value A_s^0 . Fig. 5 presents the results of this analysis for three representative values of the coupling constant, $\lambda = (5, 10, 20) \times 10^{-3}$, while keeping $g^2 = 1/2$ and $q = 2$. For each λ value, we vary χ between -30 and 0 and σ_f between 0 and σ_f^0 , where σ_f^0 is a function of χ , cf. Eq. (40). Furthermore, for each λ value, we vary ξ within a small interval, so that we cover the entire region in parameter space where the amplitude comes out close to the best-fit value A_s^0 . The lower boundaries of these intervals roughly coincide with the respective ξ values one would need in the case of single-field inflation to obtain the correct amplitude, i.e. they lie on the solid blue curve in the equivalent of Fig. 1 for $\chi = -30$. This is due to the decrease in the amplitude with decreasing $|\chi|$ as well as with decreasing σ_f/σ_f^0 , c.f. Fig. 4(b). In order to compensate for this decrease one has to employ ξ values in the two-field case that are a bit larger than in the single-field case. The resulting range of n_s values obtained for a given value of λ is marked by the shaded regions bounded by curves with a given stroke style in Fig. 5. Additionally, the solid-dashed curve marks the cosmic string bound for $\lambda = 5 \times 10^{-3}$, c.f. Eq. (52), with the region to the upper left of this curve in agreement with the bound. For the two larger values of λ , the cosmic string bound is violated in the entire χ -range shown.

The general trend in Fig. 5 is the same as in the case of single-field inflation, cf. Fig. 1: small λ values yield a large spectral index, while larger λ values give smaller n_s values. For instance, for $\lambda = 2 \times 10^{-2}$, we are able to reach n_s values below 0.98 for nearly the entire range of χ values. This illustrates that our model is in principle capable of generating a spectral index of the right magnitude, while simultaneously providing the correct amplitude of the scalar power spectrum. An obvious problem, however, is that in order to reproduce the observed amplitude A_s^0 , we require quite large ξ values, such that the cosmic string tension becomes unpleasantly large. Considering trajectories different to the σ -axis, i.e., different to the trajectory studied in Section 3, sharpens the tension imposed by the cosmic string bound, since the decrease in the amplitude due to the motion in τ -direction forces us to go to even larger values of ξ and hence larger values of $G\mu$. Moreover, we note that among the viable values for n_s

for a given value of λ and χ , the spectral index comes out smaller the closer to the σ -axis the corresponding inflationary trajectory is. In a universe undergoing a sufficiently long period of inflation, it may however not require much fine-tuning to end up with an inflationary trajectory running close to the σ -axis during the last N_* e-folds of inflation, cf. the comment below Eq. (37) in section 3.1.

5 Conclusion and outlook

Superconformal symmetry is an underlying symmetry of supergravity, broken only by fixing the value of the conformal compensator field, which generates the kinetic term of the gravitational field. It can also serve as a guideline for coupling matter fields to supergravity. The resulting supergravity models have several intriguing features. There is a Jordan frame where the Lagrangian takes a particularly simple form, closely resembling global supersymmetry. Furthermore, contrary to canonical supergravity, the scalar potential does not contain factors which grow exponentially at large field values, which keeps supergravity corrections to scalar masses under control. As we have seen, a Fayet-Iliopoulos term can be introduced analogously to the kinetic term of the graviton by making use of the conformal compensator field.

In this paper, we study hybrid models of inflation with superconformal symmetry. As we show, the inflaton acquires a large tachyonic mass in F-term hybrid inflation, which therefore is not viable. On the contrary, D-term hybrid inflation is consistent with superconformal symmetry. Allowing for an explicit symmetry breaking by a holomorphic contribution to the Kähler potential involving only dimensionless parameters [12], one obtains a two-field inflation model. If inflation lasted sufficiently long before the onset of the last 50 e-folds, the inflationary trajectory along the real part of the complex inflaton field becomes an attractor. For this limiting case we obtain analytic formulas for the amplitude of scalar fluctuations and the spectral index, which describe the full numerical results very well. It turns out that the spectral index can become as small as $n_s \simeq 0.96$. For generic two-field trajectories, we calculate the resulting amplitude of the primordial power spectrum and the spectral index numerically.

Comparing the obtained results with current CMB data, we find that for values of the gauge coupling compatible with GUTs and after fixing the overall normalization of the primordial power spectrum to the observed value, we can identify three different regions of the parameter space. For large values of the superpotential coupling λ , we

obtain a spectral index close to the current best-fit value, $n_s \simeq 0.97$. However, in this regime the model is at variance with current bounds on the cosmic string tension. On the other hand, for small values of λ , the cosmic string bound can easily be fulfilled, at the price of a 2.4σ deviation from the best-fit value for the spectral index. In the intermediate regime, the correct spectral index within 2σ experimental uncertainty can be achieved while simultaneously fulfilling the cosmic string bound.

Summarizing, superconformal D-term inflation can successfully account for the primordial power spectrum, with values of the spectral index down to $n_s \simeq 0.96$, depending on the inflationary trajectory. Generically, however, there is a tension with the cosmic string bound. This might be improved by considering a more strongly coupled theory or by considering an embedding of the simple model of D-term inflation described here into a more complete setup containing additional fields, which could yield further contributions to the primordial power spectrum. It should also be noted that the bound on the cosmic string tension contains considerable theoretical uncertainties; a better understanding of the related phenomena is necessary before parameter regions in conflict with this bound can be ruled out with certainty.

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